

the centres of the side spans. In doing so, however, the experiments show that there is a danger of the calculated speed *exceeding* the actual, whilst by taking *all* the pulleys on the two side spans into account the calculated speed will be slightly *less* than the actual speed.

If the spans of a continuous shaft, supported on bearings placed at equal distances apart, are all loaded in the same manner, each whirls independently of the rest, and the problem reduces to that of a loaded shaft supported on bearings at the ends.

The *experimental apparatus* by which the calculated results have been, for the most part, verified is shown in a figure. The experimental shaft was 2 ft. 8 in. long and 0.2488 in. diameter. The motion was transmitted from the headstock spindle to the experimental shaft by a fine piece of steel wire (about $1\frac{1}{2}$ in. long and 21 B.W.G. diam.), so that the shaft was subjected to very little constraint at the end. The experimental pulleys were models of actual pulleys—being designed for both weight and inertia. The headstock spindle was driven from a turbine, the constancy of the speed being shown by the steadiness of a column of liquid forced by a centrifugal fan indicator up a glass tube. In taking the number of revolutions corresponding to any period of whirl an ordinary counter pushed into the end of the headstock spindle was used. In making any experiment three trials were made (each of three minutes' duration) and the mean of the results taken. Over 150 experiments have been made with this apparatus, and the observed results invariably approximate very closely to the calculated results. Experiments have also been made with actual cases of shafting, and it would appear that, following the method of solution sketched above, the calculated speed is about 3 or 4 per cent. *less* than the actual speed.

The experiments were carried out in the Whitworth Engineering Laboratory, the Owens College, Manchester.

V. "On Plane Cubics." By CHARLOTTE ANGAS SCOTT, D.Sc. (Lond.), Professor of Mathematics at Bryn Mawr College, Pennsylvania. Communicated by A. R. FORSYTH, Sc.D., F.R.S. Received September 9, 1893.

(Abstract.)

In this paper the first few sections are devoted to certain constructions for the cubic, its Hessian, and its Cayleyan. Assuming three collinear inflexions for the cubic, and the tangents at these points, *i.e.*, eight conditions, one more point determines the cubic, and, consequently, also the Hessian and Cayleyan. Taking

this point on one of the known harmonic polars, the remaining two points in which the harmonic polar meets the cubic are found by a quadratic construction, and triangular symmetry completes the determination of the cubic; the inflexional tangents to the Hessian and the cusps on the Cayleyan are found by linear constructions; and the pairs of points in which these curves are met by the harmonic polar, by quadratic constructions. Any coincidence among the points so found indicates some special cubic whose properties may thus be investigated; among these special cubics are the equianharmonic cubics, whose properties present themselves very simply by means of the preliminary constructions. These special cubics are the critical ones when we follow out the variation in the Hessian and Cayleyan, which, depending directly on the variation in the original cubic, expresses itself by the change in the relative position of the points determined as above. This variation is shown in a series of diagrams exhibiting the cubic, its Hessian, and Cayleyan; and, finally, the results being compared with those derived from analysis, the variation is represented graphically by means of a single diagram.

VI. "Alternate Current Electrolysis." By J. HOPKINSON, D.Sc., F.R.S., E. WILSON, and F. LYDALL. Received November 2, 1893.

[Publication deferred.]

Presents, November 23, 1893.

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